Original Article

Institution of MECHANICAL ENGINEERS

Journal of ENGINEERING TRIBOLOGY

On lower-dimensional models in lubrication, Part A: Common misinterpretations and incorrect usage of the Reynolds equation

Proc IMechE Part J:
J Engineering Tribology
1–11

DOI: 10.1177/1350650120973792
journals.sagepub.com/home/pij
SAGE

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Abstract

Most of the problems in lubrication are studied within the context of Reynolds' equation, which can be derived by writing the incompressible Navier-Stokes equation in a dimensionless form and neglecting terms which are small under the assumption that the lubricant film is very thin. Unfortunately, the Reynolds equation is often used even though the basic assumptions under which it is derived are not satisfied. One example is in the mathematical modelling of elastohydrodynamic lubrication (EHL). In the EHL regime, the pressure is so high that the viscosity changes by several orders of magnitude. This is taken into account by just replacing the constant viscosity in either the incompressible Navier-Stokes equation or the Reynolds equation by a viscosity-pressure relation. However, there are no available rigorous arguments which justify such an assumption. The main purpose of this two-part work is to investigate if such arguments exist or not. In Part A, we formulate a generalised form of the Navier-Stokes equation for piezo-viscous incompressible fluids. By dimensional analysis of this equation we, thereafter, show that it is not possible to obtain the Reynolds equation, where the constant viscosity is replaced with a viscosity-pressure relation, by just neglecting terms which are small under the assumption that the lubricant film is very thin. The reason is that the lone assumption that the fluid film is very thin is not enough to neglect the terms, in the generalised Navier-Stokes equation, which are related to the body forces and the inertia. However, we analysed the coefficients in front of these (remaining) terms and provided arguments for when they may be neglected. In Part B, we present an alternative method to derive a lower-dimensional model, which is based on asymptotic analysis of the generalised Navier-Stokes equation as the film thickness goes to zero.

Keywords

Reynolds equation, elastohydrodynamic (or EHL), implicit constitutive relations, lower-dimensional models, piezo-viscous fluids

Date received: 16 October 2020; accepted: 19 October 2020

Introduction

It is well-known that the consequences of lubrication in e.g. bearings and gears, in terms of energy loss, materials wastage, premature failures and environmental impact are substantial, see e.g. literature.[1] To optimise the lubrication, it is necessary to have accurate and applicable mathematical models of the lubricant flow between the surfaces. We are interested in the class of lubrication problems wherein the lubricant can be considered as incompressible and piezo-viscous. This means that the Navier-Stokes consititutive relation,[a] presented here in (4), is not applicable. We will, therefore, apply the theory for implicit consititutive relations developed in literature,[2,3] and combine the implicit relation given in (10) for the lubricant rheology with conservation of mass, balance of linear and angular momentum, to model the flow. This type of system is very complex, but the fact that the fluid domain in lubrication is

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Journal of ENGINEERING TRIBOLOGY

Original Article

Proc IMechE Part J:
J Engineering Tribology
1–16
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DOI: 10.1177/1350650120973800
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On lower-dimensional models in lubrication, Part B: Derivation of a Reynolds type of equation for incompressible piezo-viscous fluids

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Abstract

The Reynolds equation is a lower-dimensional model for the pressure in a fluid confined between two adjacent surfaces that move relative to each other. It was originally derived under the assumption that the fluid is incompressible and has constant viscosity. In the existing literature, the lower-dimensional Reynolds equation is often employed as a model for the thin films, which lubricates interfaces in various machine components. For example, in the modelling of elastohydrodynamic lubrication (EHL) in gears and bearings, the pressure dependence of the viscosity is often considered by just replacing the constant viscosity in the Reynolds equation with a given viscosity-pressure relation. The arguments to justify this are heuristic, and in many cases, it is taken for granted that you can do so. This motivated us to make an attempt to formulate and present a rigorous derivation of a lower-dimensional model for the pressure when the fluid has pressure-dependent viscosity. The results of our study are presented in two parts. In Part A, we showed that for incompressible and piezo-viscous fluids it is not possible to obtain a lower-dimensional model for the pressure by just assuming that the film thickness is thin, as it is for incompressible fluids with constant viscosity. Here, in Part B, we present a method for deriving lower-dimensional models of thin-film flow, where the fluid has a pressure-dependent viscosity. The main idea is to rescale the generalised Navier-Stokes equation, which we obtained in Part A based on theory for implicit constitutive relations, so that we can pass to the limit as the film thickness goes to zero. If the scaling is correct, then the limit problem can be used as the dimensionally reduced model for the flow and it is possible to derive a type of Reynolds equation for the pressure.

Keywords

Reynolds equation, elastohydrodynamic (or EHL), implicit constitutive relations, lower-dimensional models, piezo-viscous fluids

Date received: 16 October 2020; accepted: 19 October 2020

Introduction

In this paper, we develop a lower-dimensional model that describes how a piezo-viscous fluid flows when it is confined between two adjacent surfaces that move relative to each other. The main application we have in mind is elastohydrodynamic lubrication (EHL), but the concept is generic and can be readily used to derive lower-dimensional models for various applications. This is Part B of a two-part paper. It is written to be self-contained, but to get the full picture of the content one should also read Part A of the study.

In literature,[1] Reynolds presented his famous lower-dimensional equation for the mechanical pressure p building up in a thin fluid film between two surfaces that move relative to each other. This partial differential equation, that has been named after him, is today known as the Reynolds equation and it forms the basis for most of the existing mathematical models of lubrication. The lower-dimensional model

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